

STAROBINSKY-TYPE INFLATION WITH PRODUCTS OF KÄHLER MANIFOLDS

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ABSTRACT

We present a novel realization of Starobinsky-type inflation within Supergravity using two chiral superfields. The proposed superpotential is inspired by induced-gravity models. The Kähler potential contains two logarithmic terms, one for the inflaton T and one for the matter-like field S , parameterizing the $SU(1,1)/U(1) \times SU(2)/U(1)$ Kähler manifold. The two factors have constant curvatures $-m/n$ and $2/n_2$, where n, m are the exponents of T in the superpotential and Kähler potential respectively, and $0 < n_2 \leq 6$. The inflationary observables depend on the ratio $2n/m$ only. Essentially they coincide with the observables of the original Starobinsky model. Moreover, the inflaton mass is predicted to be $3 \cdot 10^{13}$ GeV.

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1 INTRODUCTION

The clarifications regarding the impact that the dust foreground has on the observations of the B-type polarization of the CMBR, offered by the recent joint analysis of the BICEP2/*Keck Array* and *Planck* data [1, 2], revitalizes the interest in the Starobinsky model [3] of inflation. This model predicts a (scalar) spectral index $n_s \simeq 0.965$, which is in excellent agreement with observations, and a tensor-to-scalar ratio $r \simeq 0.0035$, which is significantly lower than the current upper bound. Indeed, the fitting of the data with the Λ CDM+ r model restricts [1] n_s and r in the following ranges

$$n_s = 0.968 \pm 0.0045 \quad \text{and} \quad r \lesssim 0.12 \quad (1.1)$$

at 95% *confidence level* (c.l.), with negligible n_s running: $|a_s| \ll 0.01$.

On the other hand, *Supergravity* (SUGRA) extensions of the *Starobinsky-type inflation* (STI), admit a plethora of incarnations [4–11]. In most of them two chiral superfields, T and S are employed following the general strategy introduced in Ref. [12] for the models of chaotic inflation. One prominent idea [5, 7] is, though, to parameterize with S and T the $SU(2,1)/(SU(2) \times U(1))$ Kähler manifold with constant curvature $-2/3$, as inspired by the no-scale models [13, 14]. In this context, a variety of models have been proposed in which the inflaton can be identified with either the matter-like field S [5–7] or the modulus-like field T [7–11]. We shall focus on the latter case since this implementation requires a simpler superpotential, and when connected with a MSSM version, ensures a low enough re-heating temperature, potentially consistent with the gravitino constraint [10, 15, 16].

A key issue in such SUGRA realizations of Starobinsky inflation is the stabilization of the field S accompanying the inflaton. Indeed, when the symmetry of the aforementioned Kähler manifold is respected, the inflationary path turns out to be unstable against the fluctuations of S . The instabilities can be lifted if we add to the Kähler potential K a sufficiently large quartic term $k_S |S|^4$, where $k_S > 0$ and $|k_S| \sim 1$, as suggested in Ref. [17] for models of non-minimal (chaotic) inflation [18] and applied extensively to this kind of models. This solution, however, deforms slightly the Kähler manifold [19] and is complicated to implement when more than two fields are present. In principle, all allowed quartic terms have to be considered, rendering the fluctuation analysis tedious – see e.g. Ref. [20].

Alternatively, we may utilize a nilpotent superfield S [21], or a matter field S charged under a gauged R -symmetry [19].

We propose a new solution to the stability problem that is compatible with a highly symmetric Kähler manifold. The Kähler potential involves a logarithmic function of the inflaton field T with an overall negative prefactor, as required for establishing an asymptotic inflationary plateau [7–10]. If the $|S|^2$ term is to appear in the argument of this logarithmic function, its coefficient must be negative in order to avoid negative kinetic terms. However such a negative coefficient leads to tachyonic instabilities. Therefore, we propose to split K into a sum of two logarithmic functions, one involving the inflaton field T and the other involving the field S , with negative and positive prefactors $(-n_{11})$ and n_2 , respectively. The term $|S|^2$ can now appear in the argument of the second logarithm with a positive coefficient. The prefactors $(-n_{11})$ and n_2 are selected in order to establish STI, with the field S acquiring a large enough, positive mass squared along the inflationary trajectory. The resulting Kähler potential gives rise to the product space $SU(1, 1)/U(1) \times SU(2)/U(1)$.

We would like to comment on the possibility of realizing this type of Kähler metrics in the context of string theory. The non-compact coset factor, $SU(1, 1)/U(1)$, appears in several string induced no-scale models [13, 19]. There are various classes of string inflationary models, namely D-brane inflation in warped (and unwarped) superstring compactifications, fluxbrane inflation, axion inflation, racetrack models, fibre inflation and others – see Ref. [22] for a thorough review and references therein. In models with a D-brane, there are moduli describing its position in the compactification manifold. Naively one would think that the full moduli space is a product space. The first factor, which is spanned by the brane position moduli, would be isomorphic to the internal compact manifold, and the second factor is a non-compact space spanned by the closed string moduli (such as the modulus controlling the size of the internal space). One could seek models in which the role of the inflaton is played by a closed string modulus, or as in Ref. [23] a brane position modulus. However, the stabilization of several closed string moduli requires the presence of non-trivial fluxes. And typically mixing arises between the brane position moduli with the closed string Kähler moduli – see Ref. [22, 23] for discussions. As a result, the closed string moduli space is fibered non-trivially over the space spanned by the brane position moduli, as exemplified by the DeWolfe-Giddings Kähler potential [23, 24]. If the internal, compactification manifold contains a spherical $SU(2)/U(1)$ factor, this must be supported by suitable 2-form flux, which might affect the brane worldvolume theory. Given this discussion, it may be difficult to realize a situation in which the field configuration manifold is *globally* isomorphic to the symmetric product space $SU(1, 1)/U(1) \times SU(2)/U(1)$ in the context of string inflationary models. But at least *locally* in certain regions, the moduli space could be approximated by a product space of such form. This would require to turn on suitable fluxes in order to stabilize some of the moduli in these regions. As argued in Ref. [22, 23], such a stabilization mechanism is likely to steepen the inflaton potential, halting inflation. It is thus challenging (and also interesting) to explicitly realize such a model in the context of string theory.

We implement our proposal within the framework [25–28] of *induced-gravity* (IG) models, which are generalized to highlight the robustness of our approach. The key-ingredient of our construction is the presence of the two different exponents n and m of T in the superpotential and the Kähler potential. We show that imposing a simple asymptotic condition on n, m and n_{11} , a Starobinsky-type inflationary potential gets generated, exhibiting an attractor behavior that depends only on the coefficient n_{11} , which determines the curvature of the $SU(1, 1)/U(1)$ Kähler manifold. Moreover, this model of inflation preserves a number of attractive features: (i) The superpotential and the Kähler potential may be fixed in the presence of an R -symmetry and a discrete symmetry; (ii) the initial value of the (non-canonically normalized) inflaton field can be subplanckian; (iii) the radiative corrections remain under control; and (iv) the perturbative unitarity is respected up to the reduced Planck scale [10, 26, 28, 29].

The paper is organized as follows: In Sec. 2 we generalize the formulation of STI within SUGRA IG models. In Sec. 3 we investigate totally symmetric Kähler potentials in order to find a viable inflationary scenario, which is confronted with observations in Sec. 4. Our conclusions are summarized in Sec. 5. Some mathematical notions related to the geometric structure of the Kähler manifolds encountered in our set-up are exhibited in Appendix A. Finally, Appendix B provides an analysis of the ultraviolet behavior of our models. Throughout, charge conjugation is denoted by a star (*), the symbol z as subscript denotes derivation *with respect to* (w.r.t) z and we use units where the reduced Planck scale $m_P = 2.43 \cdot 10^{18}$ GeV is equal to unity.

2 GENERALIZING THE INDUCED-GRAVITY SET-UP IN SUGRA

The realization of STI within IG models [7,8,10,27,28] requires the presence of two gauge singlet chiral superfields, the inflaton T and a “stabilizer” superfield S , which we collectively denote by z^α ($z^1 = T$ and $z^2 = S$). The relevant part of the *Einstein frame* (EF) SUGRA action is given by [18]

$$S = \int d^4x \sqrt{-\hat{g}} \left(-\frac{1}{2} \hat{\mathcal{R}} + K_{\alpha\bar{\beta}} \hat{g}^{\mu\nu} \partial_\mu z^\alpha \partial_\nu z^{*\bar{\beta}} - \hat{V} \right) \quad (2.1a)$$

where the scalar field components of the superfields z^α 's are denoted by the same superfield symbol, $K_{\alpha\bar{\beta}} = K_{,z^\alpha z^{*\bar{\beta}}}$ is the Kähler metric and $K^{\alpha\bar{\beta}}$ its inverse ($K^{\alpha\bar{\beta}} K_{\bar{\beta}\gamma} = \delta^\alpha_\gamma$). \hat{V} is the Einstein frame F-term SUGRA potential, given in terms of the Kähler potential and the superpotential W by the following expression

$$\hat{V} = e^K \left(K^{\alpha\bar{\beta}} D_\alpha W D_{\bar{\beta}} W^* - 3|W|^2 \right), \quad (2.1b)$$

where $D_\alpha W = W_{,z^\alpha} + K_{,z^\alpha} W$. Next we perform a conformal transformation [18,32] and define the *Jordan frame* (JF) metric $g_{\mu\nu}$ via the relation

$$\hat{g}_{\mu\nu} = -(\Omega/N) g_{\mu\nu}, \quad (2.2a)$$

where Ω is a frame function. In the JF, the action takes the form

$$S = \int d^4x \sqrt{-g} \left(\frac{\Omega}{2N} \mathcal{R} + \frac{3}{4N\Omega} \partial_\mu \Omega \partial^\mu \Omega - \frac{1}{N} \Omega K_{\alpha\bar{\beta}} \partial_\mu z^\alpha \partial^\mu z^{*\bar{\beta}} - V \right) \quad \text{with } V = \frac{\Omega^2}{N^2} \hat{V}. \quad (2.2b)$$

Here g stands for the determinant of $g_{\mu\nu}$; \mathcal{R} is the Ricci scalar curvature in JF, and N is a dimensionless positive parameter that quantifies the deviation from the standard set-up [18]. Let the frame function Ω and K be related by the equation

$$-\Omega/N = e^{-K/N} \Rightarrow K = -N \ln(-\Omega/N). \quad (2.3a)$$

Then using the on-shell expression [18] for the purely bosonic part of the auxiliary field

$$\mathcal{A}_\mu = i \left(K_\alpha \partial_\mu z^\alpha - K_{\bar{\alpha}} \partial_\mu z^{*\bar{\alpha}} \right) / 6, \quad (2.3b)$$

we arrive at the action

$$S = \int d^4x \sqrt{-g} \left(\frac{\Omega}{2N} \mathcal{R} + \left(\Omega_{\alpha\bar{\beta}} + \frac{3-N}{N} \frac{\Omega_\alpha \Omega_{\bar{\beta}}}{\Omega} \right) \partial_\mu z^\alpha \partial^\mu z^{*\bar{\beta}} - \frac{27}{N^3} \Omega \mathcal{A}_\mu \mathcal{A}^\mu - V \right). \quad (2.3c)$$

In terms of Ω , the auxiliary field \mathcal{A}_μ is given by

$$\mathcal{A}_\mu = -iN \left(\Omega_\alpha \partial_\mu z^\alpha - \Omega_{\bar{\alpha}} \partial_\mu z^{*\bar{\alpha}} \right) / 6\Omega \quad (2.3d)$$

where $\Omega_\alpha = \Omega_{,z^\alpha}$ and $\Omega_{\bar{\alpha}} = \Omega_{,z^{*\bar{\alpha}}}$. This last form for the JF action exemplifies the non-minimal coupling to gravity, as $-\Omega/N$ multiplies the Ricci scalar \mathcal{R} . Conventional Einstein gravity is recovered at the vacuum when

$$-\langle\Omega\rangle/N \simeq 1. \quad (2.4)$$

Starting with the JF action in Eq. (2.3c), we seek to realize STI, postulating the invariance of Ω under the action of a global \mathbb{Z}_m discrete symmetry. When S is stabilized at the origin, we write

$$-\Omega/N = \Omega_H(T) + \Omega_H^*(T^*) \quad \text{with} \quad \Omega_H(T) = c_T T^m + \sum_{k=2}^{\infty} \lambda_k T^{km}, \quad (2.5)$$

where k is a positive integer. If the values of T during inflation are subplanckian and assuming relatively low λ_k 's, the contributions of the higher powers of T in the expression above are very small, and so these can be dropped. As we will verify later, this can be achieved when the coefficient c_T is large enough. Equivalently, we may rescale the inflaton setting $T \rightarrow \tilde{T} = c_T^{1/m} T$. Then the coefficients λ_k of the higher powers in the expression of Ω get suppressed by factors of c_T^{-k} . Thus \mathbb{Z}_m and the requirement that the inflaton T is subplanckian determine the form of Ω , avoiding a severe tuning of the coefficients λ_k . Confining ourselves to such situations (and stabilizing S at the origin), Eq. (2.3a) implies that the Kähler potentials take the form

$$K_0 = -N \ln \left(f(T) + f^*(T^*) \right) \quad \text{with} \quad f(T) \simeq c_T T^m. \quad (2.6)$$

Eqs. (2.3a) and (2.4) require that T and S acquire the following vacuum expectation values

$$\langle T \rangle \simeq 1/(2c_T)^{1/m}, \quad \text{and} \quad \langle S \rangle = 0. \quad (2.7)$$

These values can be obtained, if we choose the following superpotential [27, 28]:

$$W = \lambda S \left(T^n - 1/(2c_T)^{n/m} \right), \quad (2.8)$$

since the corresponding F-term SUSY potential, V_{SUSY} , is found to be

$$V_{\text{SUSY}} = \lambda^2 \left| T^n - 1/(2c_T)^{n/m} \right|^2 + \lambda^2 n^2 |S T^{n-1}|^2 \quad (2.9)$$

and is minimized by the field configuration in Eq. (2.7). Similarly to Refs. [10, 28], we argue that when the exponent n takes integer values with $n > 1$, the form of W is constrained if we limit T to subplanckian values, and if it respects two symmetries: (i) an R symmetry under which S and T have charges 1 and 0; (ii) a discrete symmetry \mathbb{Z}_n under which only T is charged. For $n = m$, \mathbb{Z}_m becomes a symmetry of the theory and our scheme is essentially identical to those analyzed in Refs. [27, 28]. Generalizing these settings by allowing $n \neq m$, we find inflationary solutions for a variety of combinations of the parameters n, m and N – see Sec. 4.2 – including the choice $N = 3$ which appears in the no-scale SUGRA models [5, 7, 13, 14]. Note, finally, that the selected Ω in Eq. (2.5) does not contribute in the term involving Ω_{TT^*} in Eq. (2.3c). We expect that our findings are essentially unaltered even if we include in the right-hand side of Eq. (2.5) a term $-(T - T^*)^2/2N$ [27] or $-|T|^2/N$ [28] which yields $\Omega_{TT^*} = 1 \ll c_T$. In those cases, however, the symmetry of the Kähler manifolds, studied in Sec. 3, regarding the T sector of the models is violated.

The inflationary trajectory is determined by the constraints

$$S = T - T^* = 0, \quad \text{or} \quad s = \bar{s} = \theta = 0, \quad (2.10)$$

with the last equation arising when we parameterize T and S as follows

$$T = \phi e^{i\theta}/\sqrt{2}, \quad S = (s + i\bar{s})/\sqrt{2}. \quad (2.11)$$

Using the superpotential in Eq. (2.8), we find via Eq. (2.1b) that, along the inflationary path, \widehat{V} takes the following form:

$$\widehat{V}_1 = \widehat{V}(\theta = s = \bar{s} = 0) = e^K K^{SS^*} |W_{,S}|^2. \quad (2.12)$$

To identify the canonically normalized scalar fields, we cast their kinetic terms in Eq. (2.1a) into the following diagonal form

$$K_{\alpha\bar{\beta}} \dot{z}^\alpha \dot{z}^{*\bar{\beta}} = \frac{1}{2} \left(\dot{\phi}^2 + \dot{\theta}^2 \right) + \frac{1}{2} \left(\dot{\hat{s}}^2 + \dot{\hat{\bar{s}}}^2 \right), \quad (2.13a)$$

where the dot denotes derivation w.r.t the cosmic time and the hatted fields are given by

$$d\widehat{\phi}/d\phi = \sqrt{K_{TT^*}} = J, \quad \widehat{\theta} = J\theta/\phi, \quad (\widehat{s}, \widehat{\bar{s}}) = \sqrt{K_{SS^*}}(s, \bar{s}). \quad (2.13b)$$

Note that the spinor components ψ_T and ψ_S of the S and T superfields must be normalized in a similar manner, i.e., $\widehat{\psi}_S = \sqrt{K_{SS^*}}\psi_S$ and $\widehat{\psi}_T = \sqrt{K_{TT^*}}\psi_T$.

It is obvious from the considerations above, that the stabilization of S during and after inflation is of crucial importance for the realization of our scenario. This issue is addressed in the next section, where we specify the dependence of the Kähler potential on S .

3 STAROBINSKY-TYPE INFLATION & KÄHLER MANIFOLDS

We focus on Kähler potentials parameterizing totally symmetric manifolds consistent with the R symmetry acting on S . In Sec. 3.1 we review the models based on the $SU(2,1)/(SU(2) \times U(1))$ coset space. Then we analyze Kähler potentials parameterizing specific product spaces: the $SU(1,1)/U(1) \times U(1)$ space in Sec. 3.2 and the $SU(1,1)/U(1) \times SU(2)/U(1)$ space in Sec. 3.3. Among these cases, only the last one yields a satisfactory scenario.

3.1 $SU(2,1)/(SU(2) \times U(1))$ KÄHLER MANIFOLD

A typical Kähler potential employed for implementing STI in SUGRA is

$$K_1 = -n_{21} \ln \left(f(T) + f^*(T^*) - \frac{|S|^2}{n_{21}} \right), \quad (3.1)$$

with $n_{21} > 0$. The Kähler metric $K_{\alpha\bar{\beta}}$ takes the form

$$(K_{\alpha\bar{\beta}}) = m c_T e^{2K_1/n_{21}} \begin{pmatrix} m n_{21} c_T |T|^{2(m-1)} & -S T^{m-1} \\ -S^* T^{*(m-1)} & (T^m + T^{*m})/m \end{pmatrix}. \quad (3.2)$$

Using this expression, the superpotential of Eq. (2.8) and Eq. (2.1b), we obtain:

$$\begin{aligned} \widehat{V} = & \frac{\lambda^2 e^{K_1}}{(2c_T)^{2n/m} m^2 n_{21}^2 |T|^{2m}} \left(m^2 c_T^2 n_{21}^2 |T|^{2m} (T^m + T^{*m}) |f_T|^2 \right. \\ & - (2c_T)^{n/m} |S|^4 \left((2c_T)^{n/m} n |T|^{2n} (T^m + T^{*m}) + m(n_{21} - 1) (T^m T^{*n} f_T + T^{*m} T^n f_T^*) \right) \\ & + n_{21} c_T |S|^2 \left((2c_T)^{2n/m} n^2 |T|^{2n} (T^m + T^{*m})^2 + m^2 (n_{21}^2 - 3n_{21} - 1) |T|^{2m} |f_T|^2 \right. \\ & \left. \left. + (2c_T)^{n/m} n m (n_{21} - 1) (T^m + T^{*m}) (T^m T^{*n} f_T + T^{*m} T^n f_T^*) \right) \right), \end{aligned} \quad (3.3)$$

where $f_T = 1 - (2c_T)^{n/m} T^n$. Along the inflationary track in Eq. (2.10), $K_{\alpha\bar{\beta}}$ becomes diagonal

$$(K_{\alpha\bar{\beta}}) = \text{diag} \left(\frac{n_{21}m^2}{2\phi^2}, \frac{2^{n/2}}{2c_T\phi^n} \right), \quad (3.4)$$

while Eq. (3.3) reduces to Eq. (2.12), given explicitly by

$$\hat{V}_I = \frac{2^{-n+(m-2)(n_{21}-1)/2} \lambda^2 f_\phi^2}{c_T^{2n/m+n_{21}-1} \phi^{m(n_{21}-1)}} \quad \text{with} \quad f_\phi = 2^{-n/m+n/2} f_T. \quad (3.5)$$

The function f_T becomes a function of ϕ along the inflationary trajectory – see Eq. (2.11). When $c_T \gg 1$ and $\phi < 1$, or $c_T = 1$ and $\phi \gg 1$, \hat{V}_I develops a plateau with almost constant potential energy density, if the exponents are related as follows

$$2n = m(n_{21} - 1) \Rightarrow m = 2n/(n_{21} - 1). \quad (3.6)$$

For $m = n$, Eq. (3.6) yields $n_{21} = 3$, which is the standard choice – cf. Ref. [28]. Moreover, if we set $m = n = c_T = 1$ and $n_{21} = 3$, W and K_1 in Eqs. (2.8) and (3.1) yield the model of Ref. [30], which is widely employed in the literature [7–9] for implementing STI within SUGRA. As we verified numerically, the data on n_s – see Eq. (1.1) – permit only tiny (of order 0.001) deviations from Eq. (3.6), in accordance with the findings of Ref. [11]. More pronounced (of order 0.01) deviations have been found to be allowed in Ref. [31], where a higher order mixing term $|S|^2|T|^2$ is considered. In a such case, a sizable increase of r can be achieved, but the symmetry of the Kähler manifold is violated. Since integers are considered as the most natural choices for n_{21} , n and m , we adopt throughout conditions like the above one as empirical criteria for obtaining observationally acceptable STI.

Eliminating m via Eq. (3.6), \hat{V}_I and f_ϕ in Eq. (3.5) are written as – cf. Ref. [28]:

$$\hat{V}_I = \frac{2^{1-n_{21}} \lambda^2 f_\phi^2}{c_T^{2(n_{21}-1)} \phi^{2n}} \quad \text{with} \quad f_\phi = 2^{(1+n-n_{21})/2} - c_T^{(n_{21}-1)/2} \phi^n. \quad (3.7)$$

Integrating the first equation in Eq. (2.13b), we can find the EF canonically normalized field $\hat{\phi}$ as a function of ϕ . We can then express \hat{V}_I in terms of $\hat{\phi}$ obtaining

$$\hat{V}_I(\hat{\phi}) = \frac{2^{1-n_{21}} \lambda^2}{c_T^{n_{21}-1}} \left(1 - e^{-\frac{1-n_{21}}{\sqrt{2n_{21}}} \hat{\phi}} \right)^2 \quad \text{with} \quad \hat{\phi} = \frac{\sqrt{2n_{21}}}{n_{21}-1} n \ln \left((2c_T)^{(n_{21}-1)/2n} \frac{\phi}{\sqrt{2}} \right), \quad (3.8)$$

where the integration constant is evaluated so that $\hat{V}_I(\hat{\phi} = 0) = 0$. When $n = c_T = 1$ and $n_{21} = 3$, \hat{V}_I coincides with the potential extensively used in the realizations of STI. It is well-known, however, that the inflationary trajectory is unstable against the fluctuations of S [8, 18]. In Table 1, we display the mass-squared spectrum along the trajectory in Eq. (2.10) for the various choices of K . When $K = K_1$, we find $\hat{m}_s^2 < 0$, since the result is dominated by the negative term $-c_T^{n_{21}-1} \phi^{2n}$. This occurs even when $n_{21} = 1$. Note that there are no instabilities along the θ direction, since $\hat{m}_\theta^2/\hat{H}_I^2 > 1$, where $\hat{H}_I^2 = \hat{V}_I/3$ is the Hubble parameter squared, and \hat{V}_I is estimated by Eq. (3.7). In Table 1, we also list the masses $\hat{m}_{\psi_\pm}^2$ of the fermion mass-eigenstates $\hat{\psi}_\pm = (\hat{\psi}_T \pm \hat{\psi}_S)/\sqrt{2}$ given in terms of the canonically normalized spinors defined in Sec. 2.

3.2 $SU(1,1)/U(1) \times U(1)$ KÄHLER MANIFOLD

As shown in Ref. [32], in a similar set-up, the situation regarding the stability along the S direction can be improved if we choose a different Kähler potential:

$$K_2 = -n_{11} \ln \left(f(T) + f^*(T^*) \right) + |S|^2, \quad (3.9)$$

FIELDS	EIGEN- STATES	MASSES SQUARED			
			$K = K_1$	$K = K_2$	$K = K_3$
1 real scalar	$\hat{\theta}$	$\hat{m}_{\hat{\theta}}^2/\hat{H}_1^2$	$6(n_{21} - 1) \left(2^{1+n} + 2^{n_{21}} c_T^{n_{21}-1} \phi^{2n} + 2^{\frac{1}{2}(n+n_{21}-1)} (n_{21} - 5) c_T^{\frac{1}{2}(n_{21}-1)} \phi^n \right) / 2^{n_{21}} n_{21} f_{\phi}^2$	$\left(6/f_{\phi}^2 \right) (2^{n-n_{11}} + c_T^{n_{11}} \phi^{2n} + 2^{\frac{1}{2}(n-n_{11})-1} (n_{11} - 4) c_T^{n_{11}/2} \phi^n)$	
1 complex scalar	$\hat{s}, \hat{\bar{s}}$	$\hat{m}_{\hat{s}}^2/\hat{H}_1^2$	$\left(6/n_{21} f_{\phi}^2 \right) \left(2^{\frac{1}{2}(3-n_{21}+n)} c_T^{(n_{21}-1)/2} \phi^n - c_T^{n_{21}-1} \phi^{2n} + 2^{n-n_{21}} (n_{21} (n_{21} - 2) - 1) \right)$	$3 \cdot 2^{n-n_{11}} n_{11} / f_{\phi}^2$	$3 \left(2/n_2 + 2^{n-n_{11}} n_{11} / f_{\phi}^2 \right)$
2 Weyl spinors	$\hat{\psi}_{\pm}$	$\hat{m}_{\psi_{\pm}}^2$	$2^{n-2(n_{21}-1)} (n_{21} - 1)^2 \lambda^2 / n_{21} c_T^{2(n_{21}-1)} \phi^{2n}$	$2^{n-2n_{11}} n_{11} \lambda^2 / c_T^{2n_{11}} \phi^{2n}$	

Table 1: Mass-squared spectrum for $K = K_1, K_2$ and K_3 along the direction in Eq. (2.10).

where $n_{11} > 0$. This Kähler potential parameterizes [11] the $SU(1, 1)/U(1) \times U(1)$ manifold. The S field has a positive mass squared \hat{m}_s^2 , but this turns out to be less than \hat{H}_I^2 – see Table 1.

In this model the Kähler metric is diagonal for any value of T and S , i.e.,

$$(K_{\alpha\bar{\beta}}) = \text{diag} \left(\frac{n_{11}m^2|T|^{2(m-1)}}{(T^m + T^{*m})^2}, 1 \right). \quad (3.10)$$

Inserting the above result and W in Eq. (2.8) into Eq. (2.1b), we arrive at

$$\begin{aligned} \hat{V} = & \frac{\lambda^2 e^{|S|^2}}{(2c_T)^{2n/m} c_T^{n_{11}} (T^m + T^{*m})^{n_{11}}} \left((1 + |S|^2)^2 |f_T|^2 - 3|S|^2 |f_T|^2 \right. \\ & \left. + \frac{1}{m^2 n_{11}} |S|^2 |T|^{-2m} \left| mn_{11} T^m f_T + (2c_T)^{n/m} n T^n (T^m + T^{*m}) \right|^2 \right). \end{aligned} \quad (3.11)$$

Along the inflationary path, Eqs. (3.10) and (3.11) simplify as follows

$$(a) \ (K_{\alpha\bar{\beta}}) = \text{diag} \left(\frac{n_{11} m^2}{2\phi^2}, 1 \right) \quad \text{and} \quad (b) \ \hat{V}_I = \frac{2^{-n+(m-2)n_{11}/2} \lambda^2 f_\phi^2}{c_T^{2n/m+n_{11}} \phi^{mn_{11}}}, \quad (3.12)$$

where f_ϕ coincides with the function defined in Eq. (3.5), independently of n_{11} . The asymptotic condition which ensures STI is now expressed as – cf. Eq. (3.6):

$$mn_{11} = 2n \quad \Rightarrow \quad m = 2n/n_{11}. \quad (3.13)$$

As shown in Appendix A, this condition gives the ratio of the exponents m and n in terms of minus the curvature of the $SU(1, 1)/U(1)$ Kähler manifold in Planck units. For $n = m$, we end up with the IG models considered in Ref. [28] and Eq. (3.13) yields $n_{11} = 2$. Setting $n_{11} = 2(1 + \bar{n}_1)$, we find that consistency with Eq. (1.1), regarding n_s , restricts \bar{n}_1 in a very narrow region $-1/200 \lesssim \bar{n}_1 \lesssim 1/250$. Since this result indicates significant tuning, we do not pursue this possibility.

In terms of n and n_{11} , \hat{V}_I in Eq. (3.12) takes the form

$$\hat{V}_I = \frac{\lambda^2 f_\phi^2}{2^{n_{11}} c_T^{2n_{11}} \phi^{2n}} \quad \text{with} \quad f_\phi = 2^{(n-n_{11})/2} - c_T^{n_{11}/2} \phi^n. \quad (3.14)$$

As before we express ϕ and \hat{V}_I in terms of the canonically normalized field $\hat{\phi}$:

$$\hat{V}_I(\hat{\phi}) = (2c_T)^{-n_{11}} \lambda^2 \left(1 - e^{-\sqrt{n_{11}/2} \hat{\phi}} \right)^2 \quad \text{with} \quad \hat{\phi} = \sqrt{\frac{2}{n_{11}}} n \ln \left((2c_T)^{n_{11}/2n} \frac{\phi}{\sqrt{2}} \right), \quad (3.15)$$

where the integration constant satisfies the same condition as in Eq. (3.8). The resulting expressions share similar qualitative features with those expressions.

The relevant mass spectrum for the choice $K = K_2$ is shown in Table 1. Although $\hat{m}_{\chi^\alpha}^2 > 0$ for $\chi^\alpha = \theta$ and s , we observe that $\hat{m}_s^2/\hat{H}_I^2 < 1$ since $f_\phi^2 \gg 1$ for $c_T \gg 1$ and $\phi < 1$ (or $\phi \gg 1$ and $c_T < 1$). Here we take $\hat{H}_I^2 = \hat{V}_I/3$ with \hat{V}_I given by Eq. (3.14). This result arises due to the fact that only the term in the second line of Eq. (3.11) contributes to \hat{m}_s^2 . Since there is no observational hint [1] for large non-Gaussianity in the cosmic microwave background, we prefer to impose that $\hat{m}_s^2 \gg \hat{H}_I^2$ during the last 50 – 60 e-foldings of inflation. This condition guarantees that the observed curvature perturbation is generated only by ϕ , as assumed in Eq. (4.6a) below. Nonetheless, two-field inflationary models which interpolate between the Starobinsky and the quadratic model have been analyzed in Ref. [33].

3.3 $SU(1,1)/U(1) \times SU(2)/U(1)$ KÄHLER MANIFOLD

To obtain a large mass for the fluctuations of S , we replace the second factor of the product manifold of Sec. 3.2 with a compact coset space. Thus, we consider the following Kähler potential

$$K_3 = -n_{11} \ln \left(f(T) + f^*(T^*) \right) + n_2 \ln \left(1 + \frac{|S|^2}{n_2} \right), \quad (3.16)$$

where $n_2 > 0$. Eq. (3.16) together with Eqs. (2.8) and (2.1b) imply that along the inflationary direction in Eq. (2.10), $K_{\alpha\bar{\beta}}$ and \widehat{V}_1 are given by the expressions in Eq. (3.12) and $\widehat{V}_1(\widehat{\phi})$ by Eq. (3.15). Therefore, the inflationary plateau for STI is obtained by enforcing Eq. (3.13). Contrary to the model of Sec. 3.2, though, the fluctuations of S turn out to be adequately heavy, as shown in Table 1 for the choice $K = K_3$ and $0 < n_2 \leq 6$.

Indeed, $K_{\alpha\bar{\beta}}$ now differs from that in Eq. (3.10) w.r.t its second entry, i.e.,

$$(K_{\alpha\bar{\beta}}) = \text{diag} \left(\frac{n_{11}m^2|T|^{2(m-1)}}{(T^m + T^{*m})^2}, \left(1 + \frac{|S|^2}{n_2} \right)^{-2} \right). \quad (3.17)$$

Substituting $K_{\alpha\bar{\beta}}$ and W from Eq. (2.8) into Eq. (2.1b), we end up with

$$\begin{aligned} \widehat{V} &= \frac{\lambda^2 (1 + |S|^2/n_2)^{n_2}}{(2c_T)^{2n/m} c_T^{n_{11}} (T^m + T^{*m})^{n_{11}}} \left(\left(1 + \left(1 + \frac{1}{n_2} \right) |S|^2 \right)^2 |f_T|^2 - 3|S|^2 |f_T|^2 \right. \\ &\quad \left. + \frac{1}{m^2 n_{11}} |S|^2 |T|^{-2m} \left| mn_{11} T^m f_T + (2c_T)^{n/m} n T^n (T^m + T^{*m}) \right|^2 \right). \end{aligned} \quad (3.18)$$

Comparing this last expression with that in Eq. (3.11), we see that the first term in the parenthesis is enhanced by a factor $(1 + 1/n_2)$. This is the origin of the additional $6/n_2$ term in the expression of \widehat{m}_s^2 – compare in Table 1 the mass expressions for the choices $K = K_3$ and $K = K_2$. This extra term dominates when $|n_2| \leq 6$, yielding $\widehat{m}_s^2 > \widehat{H}_1^2$ (for $n_2 > 0$). On the contrary, for $n_2 < 0$ – when the corresponding Kähler manifold is $(SU(1,1)/U(1))^2$ – taking values in the range $-6 < n_2 < 0$, the instability occurring for the $K = K_1$ choice reappears. For $n_2 < -6$, the mass squared may be positive but we obtain $\widehat{m}_s^2 < \widehat{H}_1^2$, as in the $K = K_2$ case. Note that the bounds on $n_2 > 0$ constrain the curvature of the $SU(2)/U(1)$ Kähler manifold – see Appendix A. Note also that, in contrast to Eq. (3.2), the denominator of K_{TT^*} in Eq. (3.17) does not depend on S . As a consequence, no geometric destabilization [34] can be activated in our model, differently to the conventional case of STI realized by the $K = K_1$ choice.

4 INFLATION ANALYSIS

It is well known [27, 28] that STI based on \widehat{V}_1 of Eq. (3.8), with $n = m$ and $n_{21} = 3$, exhibits an attractor behavior in that the inflationary observables and the inflaton mass at the vacuum are independent of n . It would be interesting to investigate if and how this nice feature gets translated in the extended versions of STI based on \widehat{V}_1 of Eq. (3.15). In this section we examine this issue. We test our models against observations, first analytically in Sec. 4.1 and then numerically in Sec. 4.2.

4.1 ANALYTIC RESULTS

4.1.1 Duration of STI. The number of e-foldings, \widehat{N}_* , that the pivot scale $k_* = 0.05/\text{Mpc}$ undergoes during inflation has to be large enough to solve the horizon and flatness problems of the

standard Big Bag cosmology, i.e.,

$$\hat{N}_* = \int_{\hat{\phi}_f}^{\hat{\phi}_*} d\hat{\phi} \frac{\hat{V}_I}{\hat{V}_{I,\hat{\phi}}} = \int_{\phi_f}^{\phi_*} d\phi J^2 \frac{\hat{V}_I}{\hat{V}_{I,\phi}} \simeq (50 - 60). \quad (4.1)$$

The precise numerical value depends on the height of the inflationary plateau, the re-heating process and the cosmological evolution following the inflationary era [1]. Here ϕ_* [$\hat{\phi}_*$] is the value of ϕ [$\hat{\phi}$] when k_* crosses the inflationary horizon. The other integration limit, ϕ_f [$\hat{\phi}_f$], is set by the value of ϕ [$\hat{\phi}$] at the end of inflation. In the slow-roll approximation, this is determined by the condition:

$$\max\{\hat{\epsilon}(\phi_f), |\hat{\eta}(\phi_f)|\} = 1, \quad (4.2a)$$

where the slow-roll parameters, for \hat{V}_I given in Eq. (3.14), are given by – cf. Ref. [28]:

$$\hat{\epsilon} = \frac{1}{2} \left(\frac{\hat{V}_{I,\hat{\phi}}}{\hat{V}_I} \right)^2 = \frac{2^{n-n_{11}} n_{11}}{f_\phi^2} \quad \text{and} \quad \hat{\eta} = \frac{\hat{V}_{I,\hat{\phi}\hat{\phi}}}{\hat{V}_I} = \frac{n_{11}}{f_\phi^2} \left(2^{1+n-n_{11}} - 2^{\frac{1}{2}(n-n_{11})} c_T^{\frac{1}{2}n_{11}} \phi^n \right). \quad (4.2b)$$

Therefore, the end of inflation is triggered by the violation of Eq. (4.2a) at a value of ϕ given by the condition

$$\phi_f = \max \left\{ \sqrt{2} \left(\frac{1 + \sqrt{2}}{2c_T} \right)^{1/n}, 2^{(n-n_{11})/2n} \left(\frac{2 - n_{11} + \sqrt{n_{11}(n_{11} + 4)}}{2c_T^{n_{11}/2}} \right)^{1/n} \right\}. \quad (4.3)$$

The integral in Eq. (4.1) yields

$$\hat{N}_* = \frac{2^{(n_{11}-n)/2}}{n_{11}} c_T^{n_{11}/2} (\phi_*^n - \phi_f^n) - \frac{n}{n_{11}} \ln \frac{\phi_*}{\phi_f}. \quad (4.4a)$$

Ignoring the logarithmic term and taking into account that $\phi_f \ll \phi_*$, we obtain a relation between ϕ_* and \hat{N}_* :

$$\phi_* \simeq 2^{(n-n_{11})/2n} c_T^{-n_{11}/2n} \left(n_{11} \hat{N}_* \right)^{1/n}. \quad (4.4b)$$

When $c_T = 1$ the requirement of Eq. (4.1) can be fulfilled only for $\phi_* \gg 1$ – see e.g. Refs. [7–9]. On the contrary, letting c_T vary, inflation can take place with subplanckian ϕ 's, since

$$\phi_* \leq 1 \quad \Rightarrow \quad c_T \geq 2^{n/n_{11}-1} \left(n_{11} \hat{N}_* \right)^{2/n_{11}}. \quad (4.5)$$

Therefore, we need relatively large values for c_T , which increase with n and $1/n_{11}$. As shown in Appendix B, this feature of the models does not cause any problem with perturbative unitarity, since $\hat{\phi}$ in Eq. (3.15) does not coincide with ϕ at the vacuum of the theory – contrary to conventional non-minimal chaotic inflation [10, 26, 28, 29].

4.1.2 Normalization of the power spectrum. The amplitude A_s of the power spectrum of the curvature perturbation generated by ϕ at the pivot scale k_* is to be confronted with the data [1]:

$$A_s^{1/2} = \frac{1}{2\sqrt{3}\pi^3} \frac{\hat{V}_I(\hat{\phi}_*)^{3/2}}{|\hat{V}_{I,\hat{\phi}}(\hat{\phi}_*)|} = \frac{\lambda(1 - n_{11}\hat{N}_*)^2}{2^{(3+n_{11})/2} \sqrt{3}\pi c_T^{n_{11}/2} n_{11}^{3/2} \hat{N}_*} \simeq 4.627 \cdot 10^{-5}. \quad (4.6a)$$

Since the scalars listed in Table 1 for the choice $K = K_3$, with $0 < n_2 \leq 6$, are massive enough during inflation, the curvature perturbations generated by ϕ are solely responsible for generating A_s . Substituting Eqs. (4.2b) and (4.4b) into the above relation, we obtain

$$\lambda \simeq 2^{(3+n_{11})/2} \sqrt{3A_s/n_{11}} \pi c_T^{n_{11}/2} / \hat{N}_* \quad \Rightarrow \quad c_T \simeq (5.965 \cdot 10^9 \lambda^2 n_{11})^{1/n_{11}} / 2, \quad (4.6b)$$

for $\hat{N}_* \simeq 55$. Therefore, enforcing Eq. (4.6a), we obtain a constraint on $\lambda/c_T^{n_{11}/2}$ which, by virtue of Eq. (3.13), depends exclusively on n_{11} . Note, however, that c_T inherits through Eq. (4.4b) an n dependence which is also propagated to λ via Eq. (4.6b).

4.1.3 Inflationary Observables. The inflationary observables can be estimated through the relations – cf. Ref. [28]:

$$n_s = 1 - 6\widehat{\epsilon}_\star + 2\widehat{\eta}_\star = \frac{1 + n_{11}^2(\widehat{N}_\star - 2)\widehat{N}_\star - 2n_{11}(1 + \widehat{N}_\star)}{(1 - n_{11}\widehat{N}_\star)^2} \simeq 1 - \frac{2}{\widehat{N}_\star} - \frac{6}{n_{11}\widehat{N}_\star^2}, \quad (4.7a)$$

$$a_s = \frac{2}{3}(4\widehat{\eta}_\star^2 - (n_s - 1)^2) - 2\widehat{\xi}_\star = -\frac{2n_{11}^3\widehat{N}_\star(n_{11}\widehat{N}_\star + 3)}{(1 - n_{11}\widehat{N}_\star)^4} \simeq -\frac{2}{\widehat{N}_\star^2} - \frac{14}{n_{11}\widehat{N}_\star^3}, \quad (4.7b)$$

$$r = 16\widehat{\epsilon}_\star = \frac{16n_{11}}{(1 - n_{11}\widehat{N}_\star)^2} \simeq \frac{16}{n_{11}\widehat{N}_\star^2} + \frac{32}{n_{11}^2\widehat{N}_\star^3}, \quad (4.7c)$$

where $\widehat{\xi} = \widehat{V}_{1,\widehat{\phi}}\widehat{V}_{1,\widehat{\phi\phi}}/\widehat{V}_1^2$, and the variables with subscript \star are evaluated at $\phi = \phi_\star$. We observe that the analytic expressions for n_s , a_s and r depend exclusively on n_{11} , and therefore, they deviate from those obtained in Ref. [28] for the choice $K = K_1$ and $n_{21} = 3$. However, their numerical values – shown in Table 2 for $\widehat{N}_\star = 55$ and various combinations of n and n_{11} – are essentially the same with those findings. Indeed, the leading terms in the expansions in Eqs. (4.7a) and (4.7b) are identical with the corresponding ones in Ref. [28]. Only r turns out to be more sensitive to the change from n_{21} to n_{11} . In any case its value remains below 0.005 for reasonable values of n_{11} .

4.1.4 Mass of the inflaton. The EF, canonically normalized, inflaton

$$\widehat{\delta\phi} = \langle J \rangle \delta\phi \quad \text{with} \quad \langle J \rangle = \sqrt{\frac{2}{n_{11}}} \frac{n}{\langle \phi \rangle} = \frac{n}{\sqrt{n_{11}}} (2c_T)^{n_{11}/2n} \quad \text{and} \quad \delta\phi = \phi - \langle \phi \rangle \quad (4.8)$$

acquires a mass, at the SUSY vacuum – see Eq. (2.7) – given by

$$\widehat{m}_{\delta\phi} = \left\langle \widehat{V}_{1,\widehat{\phi\phi}} \right\rangle^{1/2} = \left\langle \widehat{V}_{1,\phi\phi}/J^2 \right\rangle^{1/2} = \frac{\lambda\sqrt{n_{11}}}{(2c_T)^{n_{11}/2}} \simeq \frac{2\sqrt{6A_s}\pi}{\widehat{N}_\star}. \quad (4.9)$$

Note that no SUSY breaking vacua, as those analyzed in Ref. [36], are present in our set-up. It is remarkable that $\widehat{m}_{\delta\phi}$ is essentially independent of n and n_{11} thanks to the relation between λ and c_T in Eq. (4.6b). It is also interesting that even if we had followed the same analysis for $K = K_1$ in Eq. (3.1) we would have found essentially the same mass of the inflaton. In particular in that case we would have obtained

$$\widehat{m}_{\delta\phi} = \frac{\lambda(n_{21} - 1)}{(2c_T)^{\frac{1}{2}(n_{21}-1)}\sqrt{n_{21}}} = \frac{2\sqrt{6A_s}\pi(n_{21} - 1)^4\widehat{N}_\star}{(n_{21} - (n_{21} - 1)^2\widehat{N}_\star)^2} \simeq \frac{2\sqrt{6A_s}\pi}{\widehat{N}_\star}. \quad (4.10)$$

Therefore, our models are practically indistinguishable from other versions of STI as regards $\widehat{m}_{\delta\phi}$. In other words, the condition in Eq. (3.13) generates for every n_{11} a novel – cf. Refs. [27, 28] – class of attractors in the space of the Starobinsky-like inflationary models within SUGRA.

4.2 NUMERICAL RESULTS

The analytic results presented above can be verified numerically. Let us recall that the inflationary scenario depends on the following parameters – see Eqs. (2.8) and (3.16):

$$m, n, n_{11}, n_2, c_T, \text{ and } \lambda.$$

The first three are constrained by Eq. (3.13), whereas the fourth does not affect the inflationary outputs, provided that $\widehat{m}_s^2 > \widehat{H}_I^2$ for every allowed n, n_{11} and c_T . This is satisfied when $0 < n_2 \leq 6$, as explained in Sec. 3.3. The remaining parameters together with ϕ_\star can be determined by imposing the observational constraints in Eqs. (4.1), for $\widehat{N}_\star = 55$, and (4.6a). Note that in our code we

MODEL	INPUT PARAMETERS					OUTPUT PARAMETERS				
	n	m	n_{11}	c_T	ϕ_\star	$\lambda (10^{-3})$	ϕ_f	n_s	$a_s(10^{-4})$	$r(10^{-3})$
Ceccoti-like	1	1	2	1	82	0.0018	1.7	0.965	−6.3	2.4
Dilatonic	$k/2$	k	1	1	61	0.0017	2	0.966	−6.	4.4
No-scale	$3k/2$	k	3	31	1	3.5	0.25	0.965	−6.3	1.6
IG Model With \mathbb{Z}_k	k	k	2	116	1	2	0.14	0.965	−6.3	2.4

Table 2: Input and output parameters of the models which are compatible with Eq. (4.1) for $\hat{N}_\star = 55$ and Eq. (4.6a). In cases that n and m are not specified numerically we take $k = 2$ for the computation of the parameters λ , c_T , ϕ_\star and ϕ_f .

find ϕ_\star numerically without the simplifying assumptions used for deriving Eq. (4.4b). Moreover, Eq. (4.5) bounds c_T from below, whereas Eq. (4.6a) provides a relation between c_T and λ , as derived in Eq. (4.6b). Finally, we employ the definitions of n_s , a_s and r in Eqs. (4.7a) – (4.7c) to extract the predictions of the models and Eq. (4.9) to find the inflaton mass.

In our numerical computation, we also take into account the one-loop radiative corrections, $\Delta\hat{V}_I$, to \hat{V}_I obtained from the derived mass spectrum – see Table 1 – and the well-known Coleman-Weinberg formula. It can be verified that our results are insensitive to $\Delta\hat{V}_I$, provided that the renormalization group mass scale Λ is determined by requiring $\Delta\hat{V}_I(\phi_\star) = 0$ or $\Delta\hat{V}_I(\phi_f) = 0$. A possible dependence of the results on the choice of Λ is totally avoided thanks to the smallness of $\Delta\hat{V}_I$ for any n_2 with $0 < n_2 \leq 6$, giving rise to $\Lambda \simeq (1 - 1.8) \cdot 10^{-5}$ – cf. Ref. [28]. These conclusions hold even for $\phi > 1$. Therefore, our results can be accurately reproduced by using exclusively \hat{V}_I in Eq. (3.14).

Our numerical findings for some representative values of n , m and n_{21} are presented in Table 2. In the first row we present results associated to a Ceccoti-like model [30], which is defined by $c_T = n = m = 1$. Eq. (3.13) implies that $n_{11} = 2$ and not 3 as in the original model [7, 8]. In the second and third rows we present a dilatonic and a no-scale model defined by $n_{11} = 1$ and 3, respectively. Therefore, Eq. (3.13) yields a relation between n and m . In the last row we show results concerning the IG model [27, 28] with the inflaton raised to the same exponent n in W and K_3 in Eqs. (2.8) and (3.16). In this case, Eq. (3.13) dictates that $n_{11} = 2$. The extended IG model described in Sec. 2 provides the necessary flexibility to obtain solutions to Eq. (3.13), even with $n_{11} \neq 2$, by selecting appropriately the values of n and m , as in the dilatonic and no-scale cases.

In all cases shown in Table 2, our predictions for n_s , a_s and r depend exclusively on n_{11} , and they are in excellent agreement with the analytic findings of Eqs. (4.7a) – (4.7c). On the other hand, the presented c_T , λ , ϕ_\star and ϕ_f values depend on two of the three parameters n , m and n_{11} . For the values displayed, we take $k = 2$. We remark that the resulting $n_s \simeq 0.965$ is close to its observationally central value; r is of the order 0.001, and $|a_s|$ is negligible. Although the values of r lie one order of magnitude below the central value of the present combined BICEP2/Keck Array and Planck results [2], these are perfectly consistent with the 95% c.l. margin in Eq. (1.1). In the first two models, we select $c_T = 1$ and so inflation takes place for $\phi \geq 1$ whereas for the two other cases we choose a c_T value so that $\phi_\star = 1$. Therefore, the presented c_T is the minimal one, in agreement with Eq. (4.5). Finally in all cases, we obtain $\hat{m}_{\delta\phi} \simeq 1.25 \cdot 10^{-5}$ as anticipated in Eq. (4.9).

The most crucial output of our computation is the stabilization of S (and θ) during and after inflation. To highlight further this property, we present in Fig. 1 the variations of \hat{m}_s^2/\hat{H}_I^2 and $\hat{m}_\theta^2/\hat{H}_I^2$ as functions of ϕ for the inputs shown in the two last rows of Table 2, taking $n_2 = n_{11}$ and $k = 2$. It is

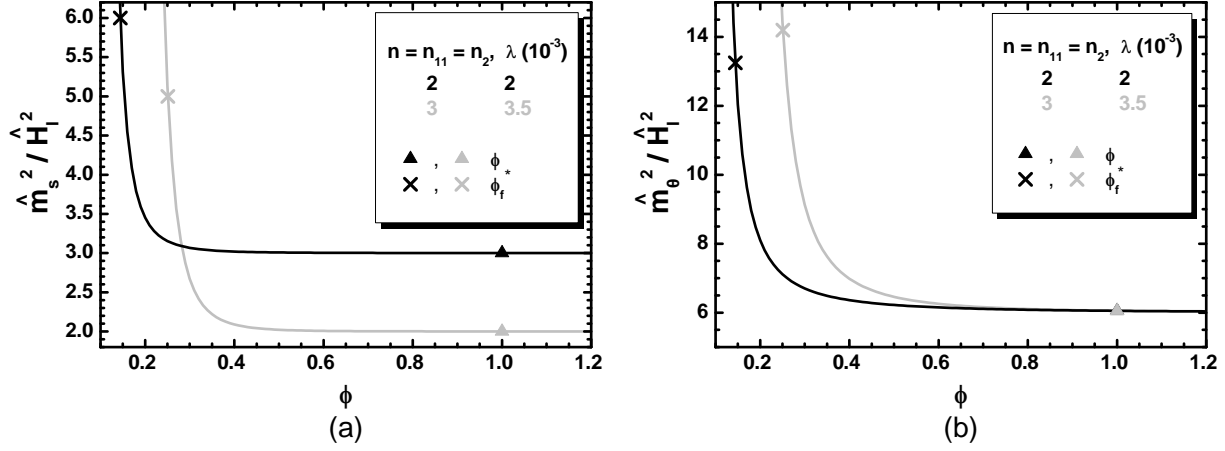


Figure 1: The ratios \hat{m}_s^2/\hat{H}_I^2 (a) and $\hat{m}_\theta^2/\hat{H}_I^2$ (b) as functions of ϕ for $n = m = n_{11} = n_2 = 2$ and $\lambda = 2 \cdot 10^{-3}$ (black lines) or $m = 2$, $n = n_2 = n_{11} = 3$ and $\lambda = 3.5 \cdot 10^{-3}$ (light gray lines). The values corresponding to ϕ_* and ϕ_f are also depicted.

evident that \hat{m}_s^2/\hat{H}_I^2 and $\hat{m}_\theta^2/\hat{H}_I^2$ remain larger than unity for $\phi_f \leq \phi \leq \phi_*$, where ϕ_* and ϕ_f are also depicted – the two ϕ_* 's are indistinguishable in Fig. 1-(b). For most ϕ values, $\hat{m}_s^2/\hat{H}_I^2 \simeq 2$ (light gray lines) or 3 (black lines) for the no-scale or the IG model with a \mathbb{Z}_2 symmetry, respectively, whereas $\hat{m}_\theta^2/\hat{H}_I^2 \simeq 6$ for both cases. Note, finally, that both \hat{m}_s^2/\hat{H}_I^2 and $\hat{m}_\theta^2/\hat{H}_I^2$ are decreasing functions of ϕ , and so if these are larger than unity for $\phi = \phi_*$, they remain so for $\phi < \phi_*$ too. This behavior is consistent with the formulae of Table 1, given that f_ϕ^2 in the denominator of $\hat{m}_{\chi_\alpha}^2$ decreases with ϕ .

5 CONCLUSIONS

We showed that Starobinsky-like inflation can be established in the context of SUGRA using the superpotential in Eq. (2.8) and the Kähler potential in Eq. (3.16), which parameterizes the product space $SU(1,1)/U(1) \times SU(2)/U(1)$. Extending previous work [27, 28], based on induced gravity, we allow for the presence of different monomials (with exponents n and m) of the inflaton superfield in W and K . Observationally acceptable inflationary solutions are attained imposing the condition in Eq. (3.13), which relates the exponents above with the curvature of the $SU(1,1)/U(1)$ space, $-2/n_{11}$. As a consequence the inflationary predictions exhibit an attractor behavior depending exclusively on n_{11} . Namely, we obtained $n_s \simeq 0.965$ and $0.001 \lesssim r \lesssim 0.005$ with negligible a_s . Moreover, the mass of the inflaton turns out to be close to $1.25 \cdot 10^{-5}$. The accompanying field S is heavy enough and well stabilized during and after inflation, provided that the curvature of the $SU(2)/U(1)$ space is such that $0 < n_2 \leq 6$. Therefore, Starobinsky inflation realized within this SUGRA setting preserves its original predictive power. Furthermore it could be potentially embedded in string theory. If we adopt $c_T \gg 1$ and $n = m > 1$, our models can be fixed if we impose two global symmetries – a continuous R and a discrete \mathbb{Z}_n symmetry – in conjunction with the requirement that the original inflaton takes subplanckian values. The one-loop radiative corrections remain subdominant and the corresponding effective theories can be trusted up to m_P .

It is argued [35] that the models described by Eq. (3.5) for $n = m$ and $n_{21} = 3$ develop one more attractor behavior towards the (n_s, r) 's encountered in the model of quadratic chaotic inflation. However, this result is achieved only for transplanckian inflaton values, without preserving the normalization of A_s in Eq. (4.6a). For these reasons we did not pursue our investigation towards this direction.

As a last remark, we would like to point out that the S -stabilization mechanism proposed in this paper has a much wider applicability. It can be employed to the models of ordinary [18] or kinetically modified [32, 37] non-minimal chaotic (and Higgs) inflation, without causing any essential alteration to their predictions. The necessary modifications are to split the relevant Kähler potential into two parts, replacing the $|S|^2$ depended part by the corresponding one included in K_3 – see Eq. (3.16) – and adjusting conveniently – as in Eq. (3.13) – the prefactor of the logarithm including the inflaton in its argument. In those cases, though, it is not clear if the part of the Kähler potential for the inflaton sector parameterizes a symmetric Kähler manifold as in the case studied here.

APPENDICES

A MATHEMATICAL SUPPLEMENT

In this Appendix we review some mathematical properties regarding the geometrical structure of the $SU(1, 1)/U(1) \times SU(2)/U(1)$ Kähler manifold. For simplicity we present the case for which $c_T = n = m = 1$ in Eqs. (3.16) and (2.8). The structure of the $SU(1, 1)/U(1)$ coset space becomes more transparent if we define [7, 14]

$$T = \frac{1}{2} \frac{1 - Z/\sqrt{n_{11}}}{1 + Z/\sqrt{n_{11}}}. \quad (\text{A.1})$$

Upon the coordinate transformation above and a Kähler transformation, the model described by the Kähler potential

$$\tilde{K}_3 = -n_{11} \ln \left(1 - \frac{|Z|^2}{n_{11}} \right) + n_2 \ln \left(1 + \frac{|S|^2}{n_2} \right) \quad (\text{A.2})$$

and the superpotential

$$\tilde{W} = W(1 + Z/\sqrt{n_{11}})^{n_{11}} \quad (\text{A.3})$$

is equivalent to the model described by Eqs. (2.8) and (3.16). The Riemannian metric associated with \tilde{K}_3 is given by

$$ds^2 = g_{11} dZ dZ^* + g_2 dS dS^*, \quad (\text{A.4})$$

having a diagonal structure with

$$g_{11} = (1 - |Z|^2/n_{11})^{-2} \quad \text{and} \quad g_2 = (1 + |S|^2/n_2)^{-2}. \quad (\text{A.5})$$

It is straightforward to show that the form of the line element in Eq. (A.4) remains invariant under the transformations

$$\frac{Z}{\sqrt{n_{11}}} \rightarrow \frac{a_1 Z/\sqrt{n_{11}} + b_1}{b_1^* Z/\sqrt{n_{11}} + a_1^*} \quad \text{and} \quad \frac{S}{\sqrt{n_2}} \rightarrow \frac{a_2 S/\sqrt{n_2} + b_2}{-b_2^* S/\sqrt{n_2} + a_2^*}, \quad (\text{A.6})$$

provided that $|a_1|^2 - |b_1|^2 = 1$ and $|a_2|^2 + |b_2|^2 = 1$. The Kähler potential \tilde{K}_3 in Eq. (A.2) remains invariant under Eq. (A.6), up to a Kähler transformation.

The transformations in Eq. (A.6) can be used to define transitive actions of the 2×2 matrices

$$U_1 = \begin{pmatrix} a_1 & b_1 \\ b_1^* & a_1^* \end{pmatrix} \quad \text{and} \quad U_2 = \begin{pmatrix} a_2 & b_2 \\ -b_2^* & a_2^* \end{pmatrix} \quad (\text{A.7})$$

on the scalar field manifolds parameterized by Z and S respectively. These matrices have the properties

$$U_1^\dagger \sigma_3 U_1 = \sigma_3 \quad \text{and} \quad U_2^\dagger U_2 = \mathbb{1} \quad \text{with} \quad \sigma_3 = \text{diag}(1, -1) \quad \text{and} \quad \mathbb{1} = \text{diag}(1, 1), \quad (\text{A.8})$$

and so, they provide representations of the $SU(1, 1)$ and $SU(2)$ groups respectively. Now U_j with $j = 1, 2$ can be written as $U_j = \tilde{U}_j H_j$ (no summation over j is applied), where the diagonal matrices $H_j = \text{diag}(e^{i\theta_j}, e^{-i\theta_j})$ stabilize the origins of the scalar field manifolds parameterized by Z and S . Thus, the scalar field manifolds are isomorphic to the coset spaces $SU(1, 1)/U(1)$ and $SU(2)/U(1)$. Notice that

$$\tilde{U}_1 = \begin{pmatrix} \alpha_1 & c_1 \\ c_1^* & \alpha_1 \end{pmatrix} \quad \text{and} \quad \tilde{U}_2 = \begin{pmatrix} \alpha_2 & c_2 \\ -c_2^* & \alpha_2 \end{pmatrix} \quad (\text{A.9})$$

with α_j real and positive, $\alpha_1^2 - |c_1|^2 = 1$ and $\alpha_2^2 + |c_2|^2 = 1$. Therefore, \tilde{U}_j with $j = 1$ and 2 define equivalent parameterizations of the coset spaces $SU(1, 1)/U(1)$ and $SU(2)/U(1)$ respectively.

Finally, applying the formula

$$R_K = g^{-3} (g_{,z} g_{,z^*} - g g_{,zz^*}) \quad (\text{A.10})$$

for $g = g_{11}$ and $z = Z$ or $g = g_2$ and $z = S$, we find that the scalar curvatures of the spaces $SU(1, 1)/U(1)$ and $SU(2)/U(1)$ are $-2/n_{11}$ and $2/n_2$ respectively.

B THE EFFECTIVE CUT-OFF SCALE

A characteristic feature of STI compared to conventional non-minimal chaotic inflation [18] is that perturbative unitarity is retained up to m_P , despite the fact that its implementation with subplanckian ϕ values requires relatively large values of c_T – see Eq. (4.5). To show that this statement holds in the context of the generalization outlined in Sec. 2, we extract the ultraviolet cut-off scale Λ_{UV} of the effective theory following the systematic approach of Ref. [29]. We focus on the second term in the right-hand side of Eq. (2.1a) for $\mu = \nu = 0$ and $\alpha = \beta = 1$, and we expand it about $\langle \phi \rangle$, given by Eq. (4.8), in terms of $\widehat{\delta\phi}$. Our result is written as

$$J^2 \dot{\phi}^2 \simeq \left(1 - \frac{\sqrt{2n_{11}}}{n} \widehat{\delta\phi} + \frac{3n_{11}}{2n^2} \widehat{\delta\phi}^2 - \frac{\sqrt{2}n_{11}^{3/2}}{n^3} \widehat{\delta\phi}^3 + \dots \right) \widehat{\delta\phi}^2, \quad (\text{B.1a})$$

where we take into account Eq. (3.13). Expanding similarly \widehat{V}_1 in Eq. (3.14) we obtain

$$\widehat{V}_1 \simeq \frac{n_{11} \lambda^2 \widehat{\phi}^2}{2^{n_{11}+1} c_T^{n_{11}}} \left(1 - \sqrt{\frac{n_{11}}{2}} \frac{n+1}{n} \widehat{\delta\phi} + n_{11} (1+n) \frac{11+7n}{24n^2} \widehat{\delta\phi}^2 - \dots \right). \quad (\text{B.1b})$$

Since the coefficients in the series above are independent of c_T and of order unity for reasonable n and n_{11} values, we infer that our models do not face any problem with perturbative unitarity up to m_P .

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